CUET Mathematics Solved Paper-2023

Held on 25 May 2023, (Shift-I)

SECTION: COMMON

1. The degree of the differential equation

$$\left(1+\frac{dy}{dx}\right)^4=\left(\frac{d^2y}{dx^2}\right)^2 \text{ is:}$$

(a) 1 (b) 3
(c) 2 (d) 4

2. The solution of a LPP with basic feasible solutions (0, 0), (10, 0), (0, 20) (10, 15) and objective function Max $Z=2x+3y$ is:

(a) $x=0, y=20$, Max $Z=60$
(b) $x=10, y=15$, Max $Z=65$
(c) $x=10, y=20$, Max $Z=70$
(d) $x=15, y=10$, Max $Z=60$

3. If $\begin{vmatrix} 2 & 3-x \\ x & 1 \end{vmatrix}=0$, then the values of x are :

(a) 1 and 3 (b) 1 and 2
(c) 2 and 3 (d) 3 and 0

4. In a meeting, 70% of the members favour and 30% oppose a certain proposal. A member is selected at random and we take X= 0 if he opposed, and X= 1 if he is favour. Then, E (X) is:

(a) $\frac{7}{10}$ (b) $\frac{1}{2}$
(c) $\frac{1}{3}$ (d) $\frac{7}{11}$

5. If $y=\frac{1}{x+1}$, then $\frac{d^2y}{dx^2}$ at $x=2$ is:

(a) $\frac{2}{9}$ (b) $\frac{3}{2}$
(c) $\frac{2}{27}$ (d) $\frac{3}{8}$

6. If $f(x)=\frac{1}{1-x}$, then for $x>1, f(x)$ is:

(a) decreasing (b) constant
(c) increasing (d) neither decreasing nor increasing

7. $\int_0^{1.5}[x]dx$, where [x] denotes the greatest integer function $\leq x$, is equal to:

(a) $\frac{1}{4}$ (b) $\frac{1}{2}$
(c) 1 (d) 0

8. In a box containing 100 bulbs, 10 are defective. Then the probability, that out of a sample of 5 bulbs none is defective, is:

(a) 10^{-1} (b) $\left(\frac{1}{2}\right)^5$
(c) $\left(\frac{9}{10}\right)^5$ (d) $\left(\frac{9}{10}\right)$

9. Match List-I with List-II

	List-I		List-II
A.	Maximum value of $f(x)=-\lvert x+1\rvert+3$	I.	6
B.	Minimum value of $f(x)=(2x-1)^2+5$	II.	5
C.	Maximum value of $f(x)=6-x^2$	III.	no maximum value
D.	Maximum value of $f(x)=x^3+1$	IV.	3

Choose the correct answer from the options given below:

(a) A-IV, B-II, C-I, D-III (b) A-III, B-IV, C-I, D-II
(c) A-I, B-II, C-III, D-IV (d) A-II, B-III, C-IV, D-I

10. The differential equation $\frac{dy}{dx}+\frac{x}{y}=0$, represents the family of curves:

(a) $x^2-y^2=C$ (b) $\frac{x}{y}=C$

(c) $xy=C$ (d) $x^2+y^2=C$

11. If matrix A is of order 2×3 and B of order 3×2, then

(a) AB, BA both are defined and are equal

(b) AB is defined but BA is not defined

(c) AB is not defined but BA is defined

(d) AB, BA are defined but are not equal

12. The area enclosed by the ellipse $\frac{x^2}{9^2}+\frac{y^2}{6^2}=1$ is:

(a) 15π (b) 54π

(c) 18π (d) $\frac{3}{2}\pi$

13. The programming problem Max $Z=2x+3y$ subject to the conditions $0\le x\le 3, 0\le y\le 4$ is:

(a) not an LPP

(b) an LPP, with unbounded feasible region and no solution

(c) an LPP, and Max Z = 18, at $x=3, y=4$

(d) an LPP, and Max Z = 12, at $x=0, y=4$

14. If A is a square matrix of order 3, B = kA and $|B|=x|A|$ then,

(a) $x=2k$ (b) $x=k^2$

(c) $x=k^3$ (d) $x=3k$

15. The matrix $A=\begin{bmatrix}0 & 1 & -3\\ -1 & 0 & 0\\ 3 & 0 & 0\end{bmatrix}$ is a

(a) Diagonal matrix

(b) Symmetric matrix

(c) Skew-symmetric matrix

(d) Scalar matrix

SECTION: CORE MATHEMATICS

1. Match List I with List II

List-I	List-II
A. The area of parallelogram determined by vectors $2\hat{i}$ and $3\hat{j}$	I. 2
B. The value of $(\hat{i}\times\hat{j})\cdot\hat{k}+(\hat{j}\times\hat{k})\cdot\hat{i}$	II 4
C. The value of a for which the vectors $2\hat{i}-3\hat{j}+4\hat{k}$ and $a\hat{i}-6\hat{j}+8\hat{k}$ are collinear	III 0
D. The value of λ for which the vectors $2\hat{i}+\hat{j}+\hat{k}$ and $2\hat{i}-4\hat{j}+\lambda\hat{k}$ are perpendicular	IV. 6

Choose the correct answer from the options given below:

(a) A-I, B-II, C-III, D-IV (b) A-II, B-I, C-III, D-IV

(c) A-III, B-IV, C-II, D-I (d) A-IV, B-I, C-II, D-III

2. The derivative of $\sin(\tan^{-1}e^{2x})$ with respect to x is:

(a) $\frac{2e^{2x}\sin(\tan^{-1}e^{2x})}{1+e^{4x}}$ (b) $\frac{2e^{2x}\cos(\tan^{-1}e^{2x})}{1+e^{4x}}$

(c) $\frac{2e^{2x}\sin(\tan^{-1}e^{2x})}{1+e^{x^2}}$ (d) $\frac{2e^{2x}\cos(\tan^{-1}e^{2x})}{1+e^{2x}}$

3. If the matrix $A=\begin{bmatrix}0 & x+y & 1\\ 3 & z & 2\\ x-y & -2 & 0\end{bmatrix}$ is skew symmetric, then:

(a) $x=2, y=1, z=0$ (b) $x=2, y=2, z=0$

(c) $x=-2, y=-1, z=0$ (d) $x=-2, y=-1, z=-1$

4. Particular solution of the differential equation $\log\left(\frac{dy}{dx}\right)=x+y$, given that when $x=0, y=0$ is:

(a) $e^x+e^{-y}=2$ (b) $e^{-x}+e^y=2$

(c) $e^x+e^y=2$ (d) $e^{-x}+e^{-y}=2$

5. If A is a square matrix of order 3, then |adj A| is equal to:

(a) $|A|$ (b) $|A|^2$

(c) $|A|^3$ (d) $3|A|$

6. Two dice are thrown simultaneously. If X denotes the number of sixes, then the variance of X is:

(a) $\frac{5}{18}$ (b) $\frac{7}{18}$

(c) $\frac{1}{3}$ (d) $\frac{2}{3}$

7. If three points $A(a_1, b_1)$, $B(a_2, b_2)$ and $C(a_3, b_3)$ are collinear and $D = \begin{vmatrix} a_1 & b_1 & 1 \\ a_2 & b_2 & 1 \\ a_3 & b_3 & 1 \end{vmatrix}$, then:

(a) $D = 0$

(b) $D = \pm 1$

(c) $D^2 = 0$ or 1

(d) $D = (a_1 + a_2 + a_3) - (b_1 + b_2 + b_3)$

8. The area of the region bounded by the lines $x = 2y + 3$, $x = 0$, $y = 1$ and $y = -1$ is:

(a) 4 sq . units (b) 6 sq . units

(c) 8 sq . units (d) $\frac{3}{2}$ sq . units

9. The feasible region of an LPP Max $Z = 3x + 2y$ subject to $x \geq 0, y \geq 0, x - 2y \leq 3$ is:

(a) Bounded in first quadrant but has no solution

(b) Unbounded in first quadrant but has a solution

(c) Unbounded in first quadrant and has no solution

(d) Bounded and has a solution $x = 0, y = 0, z = 0$

10. The function $f(x) = \frac{x-1}{x(x^2-1)}, x \neq 1, f(1) = 1$, is discontinous at

(a) Exactly one point (b) Exactly two points

(c) Exactly three points (d) No point

11. Match List I with List II

List I		List II	
A.	$\sin^{-1}x + \cos^{-1}x$, $x \in [-1,1]$	I.	$-\frac{\pi}{2}$
B.	$\tan^{-1}\sqrt{3} - \cot^{-1}(-\sqrt{3})$	II.	$-\frac{\pi}{6}$
C.	$\cos^{-1}\left(\cos\frac{13\pi}{6}\right)$	III.	$\frac{\pi}{2}$
D.	$\sin^{-1}\left(-\frac{1}{2}\right)$	IV.	$\frac{\pi}{6}$

Choose the correct answer from the options given below:

(a) A-III, B-I, C-IV, D-II (b) A-IV, B-I, C-II, D-III

(c) A-II, B-III, C-IV, D-I (d) A-I, B-II, C-III, D-IV

12. Probabilities to solve a specific problem by A, B and C are $\frac{1}{2}, \frac{1}{3}$ and $\frac{1}{4}$ respectively. Probability that at least one will solve the problem is:

(a) $\frac{1}{24}$ (b) $\frac{1}{4}$

(c) $\frac{23}{24}$ (d) $\frac{3}{4}$

13. Let $A = \{1, 2, 3\}$. Consider the relation $R = \{(1,1), (2, 2), (3, 3), (1, 2), (2, 3), (1, 3)\}$. Then R is

(a) reflexive only

(b) reflexive and transitive

(c) symmetric and transitive

(d) neither symmetric nor transitive

14. Solution of $\frac{dy}{dx} = (1+x^2)(1+y^2)$ is:

(a) $\tan^{-1} y = x + \frac{x^3}{3} + c$

(b) $\tan^{-1} y = x - \frac{x^3}{3} + c$

(c) $\tan^{-1} y = x^2 + \frac{x^3}{3} + c$

(d) $\tan^{-1} y = x^2 - \frac{x^3}{3} + c$

15. If $\sqrt{1-x^2} + \sqrt{1-y^2} = a(x-y)$, then $\frac{dy}{dx} =$

(a) $\sqrt{\frac{1-x^2}{1-y^2}}$ (b) $\sqrt{\frac{1-y^2}{1-x^2}}$

(c) $\sqrt{\frac{1-x^2}{1+y^2}}$ (d) $\sqrt{\frac{1+x^2}{1-y^2}}$

16. If a line makes angles 90°, 60° and θ with x, y and z axis respectively, where θ is acute, then value of θ is:

(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{4}$

(c) $\frac{\pi}{3}$ (d) $\frac{\pi}{2}$

17. The area of the region bounded by the parabola $y^2 = 4ax$ and its latus rectum is:

(a) $\frac{4a^2}{3}$ sq. units (b) $\frac{8a^2}{3}$ sq. units

(c) $\frac{2a^2}{3}$ sq. units (d) $\frac{9a^2}{5}$ sq. units

18. If $A=\begin{bmatrix}\cos\theta & \sin\theta & 0\\ -\sin\theta & \cos\theta & 0\\ 0 & 0 & 1\end{bmatrix}$ and B is a square matrix of order 3, then |AB| is equal to:

(a) $|B|^2$ (b) $|B|$

(c) $\sin^2\theta|B|$ (d) $\cos^2\theta|B|$

19. If $f: R \to R$ is defined by $f(x) = \sin x + x$, then $f(f(x))$ is:

(a) $2\sin x + 2x$ (b) $\sin^2 x + x^2$

(c) $\sin(\sin x + x) + \sin x + x$ (d) $\sin^2 x + 2\sin x + x$

20. $\int\left(\frac{1+x+x^2}{1+x^2}\right)e^{\tan^{-1}x}dx =$

(a) $x + e^{\tan^{-1}x} + c$ (b) $e^{\tan^{-1}x} - x + c$

(c) $e^{\tan^{-1}x} + c$ (d) $xe^{\tan^{-1}x} + c$

21. A manufacturer can sell x items at a price of ₹ 3x + 5 each. The cost price of x items is ₹ $x^2 + 5x$. If x is the number of items she should sell to get no profit and no loss, then:

(a) x = 10 (b) x = 30

(c) x = 0 (d) x = −10

22. The angle between the line $\frac{x+2}{3} = \frac{y-3}{2} = \frac{z+5}{6}$ and the plane $2x + 10y - 11z = 5$ is:

(a) $\cos^{-1}\left(\frac{8}{21}\right)$ (b) $\sin^{-1}\left(\frac{8}{21}\right)$

(c) $\cos^{-1}\left(\frac{21}{82}\right)$ (d) $\sin^{-1}\left(\frac{21}{82}\right)$

23. A. Equation of the line passing through the point (1, 2, 3) and parallel to the vector $3\hat{i} + 2\hat{j} - 2\hat{k}$ is $\frac{x-1}{3} = \frac{y-2}{2} = \frac{z-3}{-2}$

B. Equation of line passing through (1, 2, 3) and parallel to the line given by $\frac{x+3}{3} = \frac{4-y}{5} = \frac{z+8}{6}$ is $\frac{x-1}{3} = \frac{y-2}{5} = \frac{z+3}{6}$

C. Equation of line passing through the origin and (5, −2, 3) is $\frac{x}{5} = \frac{y}{-2} = \frac{z}{3}$

D. Equation of plane passing through the point (1, 2, 3) and perpendicular to the line with direction ratio's 2, 3, −1 is $2(x-1) + 3(y-2) - 1(z-3) = 0$.

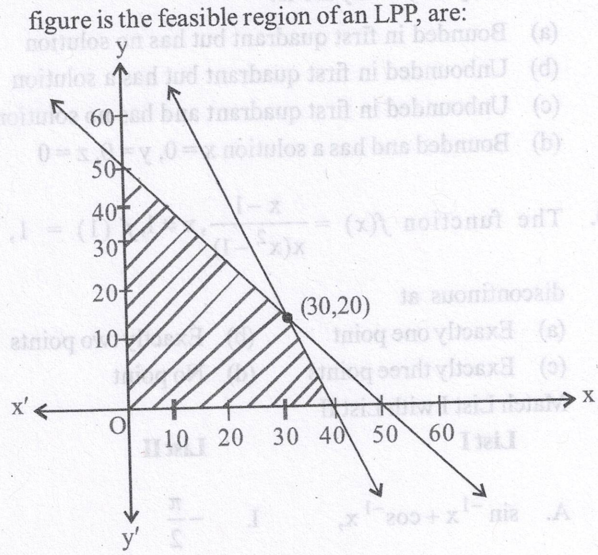
E Equation of plane with intercepts 2, 3 and 4 on x, y and z-axis respectively is $2x + 3y + 4z = 1$.

Choose the correct answer from the options given below:

(a) A, E only (b) A, C, D only

(c) C, D, E only (d) E only

24. The linear constraints, for which the shaded area in the figure is the feasible region of an LPP, are:



(a) $x + y \ge 50$, $2x + y \le 80$, $x, y \ge 0$

(b) $x + y \le 50$, $2x + y \ge 80$, $x, y \ge 0$

(c) $x + y \le 50$, $2x + y \le 80$, $x, y \ge 0$

(d) $x + y \ge 50$, $2x + y \ge 80$, $x, y \ge 0$

25. The approximate volume of a cube of side a metres on increasing the side by 4% is:

(a) $1.04a^3\ m^3$ (b) $1.004\ a^3\ m^3$

(c) $1.12\ a^3\ m^3$ (d) $1.12\ a^2\ m^3$

26. The maximum slope of the curve $y = -x^3 + 3x^2 + 9x - 27$ is:

(a) 0 (b) 12

(c) 16 (d) 32

27. Match List I with List II

List-I	List-II
A. $\int \frac{\sin x}{1+\cos x}dx$	I. $e^{\tan^{-1}x}+C$
B. $\int \frac{1}{1-\tan x}dx$	II. $\log(\log x+1)+C$
C. $\int \frac{e^{\tan^{-1}x}}{1+x^2}dx$	III. $-\log\|1+\cos x\|+C$
D. $\int \frac{1}{x+x\log x}dx$	IV. $\frac{x}{2}-\frac{1}{2}\log\|\cos x-\sin x\|+C$

Choose the correct answer from the options given below:

(a) A-II, B-III, C-IV, D-I (b) A-III, B-IV, C-I, D-II

(c) A-I, B-II, C-III, D-IV (d) A-IV, B-I, C-III, D-II

28. Which of the following statements is NOT CORRECT.

(a) A row matrix has only one row.

(b) A diagonal matrix has all diagonal elements equal to zero

(c) A symmetric matrix is a square matrix satisfying certain conditions.

(d) A skew- symmetric matrix has all diagonal elements equal to zero.

29. If the matrix $A=\begin{bmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{bmatrix}$, then A^2 is equal to:]

(a) $\begin{bmatrix}\cos 2\theta & \sin 2\theta\\ -\sin 2\theta & \cos 2\theta\end{bmatrix}$

(b) $\begin{bmatrix}\cos^2\theta & \sin^2\theta\\ -\sin^2\theta & \cos^2\theta\end{bmatrix}$

(c) $\begin{bmatrix}\cos\theta^2 & \sin\theta^2\\ -\sin\theta^2 & \cos\theta^2\end{bmatrix}$

(d) $\begin{bmatrix}\cos\theta+\sin\theta & \cos\theta-\sin\theta\\ \sin\theta-\cos\theta & \cos\theta+\sin\theta\end{bmatrix}$

30. Value of $e^{\frac{e^{\sin(\tan^{-1}x+\cot^{-1}x)}}{\sin(\sin^{-1}x+\cos^{-1}x)}}$, $x\in[-1,1]$, is:

(a) 0 (b) $\frac{\pi}{2}$

(c) 1 (d) $-\frac{\pi}{2}$

31. The range of the function $f(x)=\frac{1}{3-\sin 4x}$ is:

(a) $\left[\frac{1}{4},\frac{1}{2}\right]$ (b) $\left[\frac{1}{2},1\right]$

(c) $\left[\frac{1}{4},\frac{3}{4}\right]$ (d) $\left[\frac{1}{2},\frac{3}{4}\right]$

32. The equation of the tangent to the curve $y=x^2-2x-3$ which is perpendicular to the line $x+2y+3=0$, is

(a) $4x-2y=7$ (b) $2x-y=7$

(c) $2x-y=5$ (d) $4x-2y=5$

33. The solution of the differentiable equation $2x\frac{dy}{dx}+y=14x^3, x>0$, is

(a) $y=2x^3+cx^{\frac{1}{2}}$ (b) $y=x^3+cx^{\frac{1}{2}}$

(c) $y=2x^3+cx^{-\frac{1}{2}}$ (d) $y=x^3+cx^{-\frac{1}{2}}$

34. Let the vectors $\vec{a}=\hat{i}-3\hat{j}+2\hat{k}, \vec{b}=2\hat{i}+\hat{j}-\hat{k}$ and $\vec{c}=3\hat{i}+5\hat{j}-2\lambda\hat{k}$ be coplanar. Then λ is equal to

(a) –1 (b) 1

(c) –2 (d) 2

35. A coin is tossed 7 times. The probability of getting at least 4 heads is:

(a) $\frac{5}{8}$ (b) $\frac{3}{4}$

(c) $\frac{1}{4}$ (d) $\frac{1}{2}$

Hints & Explanations

1. (c) Since highest order term have power 2, hence degree of the given differential equation = 2.

2. (d) Given $Z = 2x + 3y$

At point (0, 0), $Z_{(0,0)} = 0$

At point (10, 0), $Z_{(10,0)} = 20$

At point (10, 15), $Z_{(10,15)} = 65$

(which is maximum value of z)

3. (b) Given $\begin{vmatrix} 2 & 3-x \\ x & 1 \end{vmatrix} = 0$

$\Rightarrow 2 - x(3-x) = 0$

$\Rightarrow x^2 - 3x + 2 = 0$

$\Rightarrow x = 1, 2.$

4. (a)

X	0	1	*Total*
$P(X)$	$\frac{30}{100}$	$\frac{70}{100}$	1
$X\,P(X)$	0	$\frac{70}{100}$	$\frac{70}{100}$

$\therefore E(X) = \frac{70}{100} = \frac{7}{10}.$

5. (c) $y = \frac{1}{(x+1)}$

$\Rightarrow \frac{dy}{dx} = -\frac{1}{(x+1)^2} \Rightarrow \frac{d^2y}{dx^2} = \frac{2}{(x+1)^3}$

$\Rightarrow \left(\frac{d^2y}{dx^2}\right)_{x=2} = \frac{2}{(2+1)^3} = \frac{2}{27}.$

6. (c) $f(x) = \frac{1}{(1-x)} \Rightarrow f'(x) = \frac{1}{(1-x)^2}$

Since $f'(x)$ is positive $\forall\ x > 1$

$\Rightarrow f'(x)$ is increasing for $\forall\ x > 1$

7. (b) $\int_0^{1.5} [x]\,dx = \int_0^1 (0)\,dx + \int_1^{1.5} (1)\,dx$

$= \int_1^{1.5} dx = [x]_1^{1.5} = 1.5 - 1 = \frac{1}{2}.$

8. (c) $n = 5$

X (No. of defective bulbs) = 0, 1, 2, 3, 4, 5

$p = \frac{10}{100} = \frac{1}{10}$ and $q = 1 - \frac{1}{10} = \frac{9}{10}$

$\therefore p$ (none defection) $= p(X = 0)$

$= {}^5C_0\left(\frac{1}{10}\right)^0\left(\frac{9}{10}\right)^5 = \left(\frac{9}{10}\right)^5$

9. (a)

(A) $0 \le |x+1| < \infty \Rightarrow -\infty < -|x+1| \le 0$

$\Rightarrow -\infty < -|x+1| + 3 \le 3$

$\Rightarrow -\infty < f(x) \le 3$

$\Rightarrow$ Maximum value of $f(x)$ is 3 $\Rightarrow$ (A $\to$ IV)

(B) $0 \le (2x-1)^2 < \infty \Rightarrow 5 \le (2x-1)^2 + 5 < \infty$

$\Rightarrow$ Minimum value is 5 $\Rightarrow$ (B $\to$ II)

(C) $0 \le x^2 < \infty \Rightarrow -\infty < -x^2 \le 0 \Rightarrow -\infty < 6 - x^2 \le 6$

$\Rightarrow$ Maximum value is 6 $\Rightarrow$ (C $\to$ I)

(D) $-\infty < x < \infty \Rightarrow -\infty < x^3 < \infty \Rightarrow -\infty < x^3 + 1 < \infty$

$\Rightarrow f(x)$ have no maximum value $\Rightarrow$ (D $\to$ III)

10. (d) $\frac{dy}{dx} + \frac{x}{y} = 0 \Rightarrow y\,dy + x\,dx = 0$

$\Rightarrow \int y\,dy + \int x\,dx = 0$

$\Rightarrow \frac{1}{2}y^2 + \frac{1}{2}x^2 = \frac{C}{2}$

$\Rightarrow x^2 + y^2 = C.$

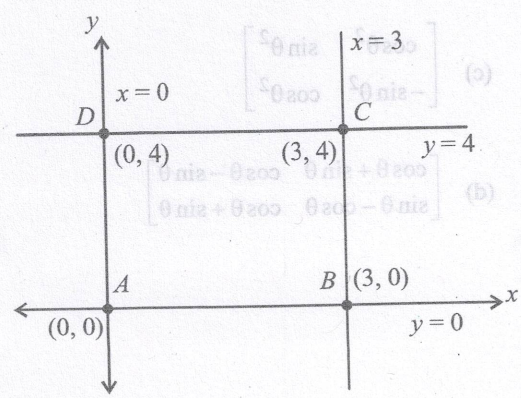
11. (d) Matrix multiplication $A_{m\times n} \cdot B_{p\times q}$ is possible iff $p = m$. Hence $A_{2\times 3} \cdot B_{3\times 2}$ and $B_{3\times 2} \cdot A_{2\times 3}$ both are defined but it is not necessary that AB and BA should be equal.

12. (b) Area enclosed by the ellipse

$= \pi$ (major axis) (minor axis)

$= \pi\,(9)\,(6) = 54\pi.$

13. (c) Given $Z = 2x + 3y$



Rectangle $ABCD$ is the bounded feasible region.

At $A(0, 0)$, $Z = 0$

At $B(3, 0)$, $Z = 6$

At $C(3, 4)$, $Z = 18$ (which is max. of Z)

At $D(0, 4)$, $Z = 12$

14. (c) Given $|B| = x|A|$

$\Rightarrow |KA| = x|A| \quad (\because B = KA)$

$\Rightarrow K^3|A| = x|A| \quad (\because A$ is square matrix of order 3)

$\Rightarrow x = K^3$.

15. (c) $A = \begin{bmatrix} 0 & 1 & -3 \\ -1 & 0 & 0 \\ 3 & 0 & 0 \end{bmatrix} \Rightarrow A^T = \begin{bmatrix} 0 & -1 & 3 \\ 1 & 0 & 0 \\ -3 & 0 & 0 \end{bmatrix} = -A$

$\Rightarrow A$ is skew symmetric matrix.

SECTION : CORE MATHEMATICS

1. (d)

(A) Area of Parallelogram $= |2\hat{i} \times 3\hat{j}| = |6\hat{k}| = 6$

(A $\rightarrow$ IV)

(B) $(\hat{i} \times \hat{j}) \cdot \hat{k} + (\hat{j} \times \hat{k}) \cdot \hat{i} = \hat{k} \cdot \hat{k} + \hat{i} \cdot \hat{i} = 1 + 1 = 2$

(B $\rightarrow$ I)

(C) Since $(a\hat{i} - 6\hat{j} + 8\hat{k})$ and $(2\hat{i} - 3\hat{j} + 4\hat{k})$ are collinear.

Hence, $\frac{a}{2} = \frac{-6}{-3} \Rightarrow a = 4$ (C $\rightarrow$ II)

(D) Since $(2\hat{i} + \hat{j} + \hat{k})$ and $(2\hat{i} - 4\hat{j} + \lambda\hat{k})$ are perpendicular.

Hence $(2\hat{i} + \hat{j} + \hat{k}) \cdot (2\hat{i} - 4\hat{j} + \lambda\hat{k}) = 0$

$\Rightarrow 4 - 4 + \lambda = 0 \Rightarrow \lambda = 0$ (D $\rightarrow$ III)

2. (b) Let $y = \sin(\tan^{-1} e^{2x})$

$$\Rightarrow \frac{dy}{dx} = \frac{\cos(\tan^{-1} e^{2x}) \cdot e^{2x} \cdot 2}{(e^{2x})^2 + 1}$$

$$\Rightarrow \frac{dy}{dz} = \frac{2e^{2x} \cos(\tan^{-1} e^{2x})}{1 + e^{4x}}.$$

3. (c) Since A is skew symmetric $\Rightarrow A^T = -A$.

$$\Rightarrow \begin{bmatrix} 0 & 3 & (x-y) \\ (x+y) & z & -2 \\ 1 & 2 & 0 \end{bmatrix} = -\begin{bmatrix} 0 & x+y & 1 \\ 3 & z & 2 \\ x-y & -2 & 0 \end{bmatrix}$$

by comparison we get, $x + y = -3$ and $x - y = -1$

$\Rightarrow x = -2, y = -1$

Since A is skew symmetric

$\Rightarrow$ diagonal element will be zero

$\Rightarrow z = 0$.

4. (a) $\log\left(\frac{dy}{dx}\right) = x + y \Rightarrow \frac{dy}{dx} = e^{x+y} = e^x \cdot e^y$

$\Rightarrow e^{-y}\, dy = e^x\, dx \Rightarrow -e^{-y} = e^x + C$

When $x = y = 0$ then $-1 = 1 + C \Rightarrow C = -2$

Hence particular solution is $e^x + e^{-y} = 2$.

5. (b) If $A_{n \times n}$ is invertible matrix then $|\text{adj } A| = |A|^{n-1}$.

$\Rightarrow |\text{adj } A| = |A|^{3-1} = |A|^2 \ (\because n = 3)$

6. (a) $n = 2$

X (No. of sines) = 0, 1, 2

$p = \frac{1}{6}, q = \frac{5}{6}$

Variance $= npq = 2 \times \frac{1}{6} \times \frac{5}{6} = \frac{5}{18}$.

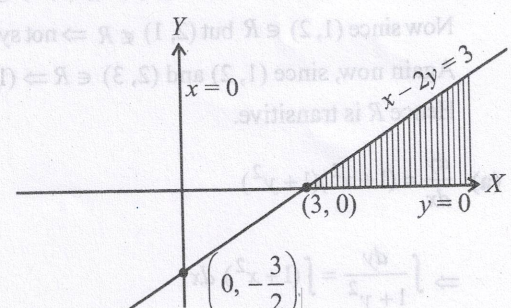
7. (a) If $A(a_1, b_1), B(a_2, b_2)$ and $C(a_3, b_3)$ are collinear then

$$\begin{vmatrix} a_1 & b_1 & 1 \\ a_2 & b_2 & 1 \\ a_3 & b_3 & 1 \end{vmatrix} = 0 \Rightarrow D = 0.$$

8. (b) Required Area $= \int_{y=-1}^{y=1} x\, dy = \int_{y=-1}^{y=1} (2y + 3)\, dy$

$$= \left[y^2 + 3y\right]_{y=-1}^{y=1} = (1 + 3) - (1 - 3) = 6.$$

9. (c) Plotting given constraints, we get a unbounded region in first quadrant.



Clearly Max $z = 3x + 2y$ has no solution in the shaded region.

10. (a) RHL $= \lim_{h \to 0} f(1+h) = \lim_{h \to 0} \frac{(1+h) - 1}{(1+h)[(1+h)^2 - 1]}$

$$\Rightarrow \text{RHL} = \lim_{h \to 0} \frac{h}{(1+h)\,h(h+2)} = \lim_{h \to 0} \frac{1}{(1+h)(h+2)}$$

$\Rightarrow$ RHL $= \frac{1}{2} \neq f(1) \Rightarrow f(x)$ is discontinuous at $x = 1$

$\Rightarrow f(x)$ is discontinuous exactly at one point.

11. (a)

(A) $\sin^{-1} x + \cos^{-1} x = \frac{\pi}{2} \Rightarrow (A \to III)$

(B) $\tan^{-1}\sqrt{3} - \cot^{-1}(-\sqrt{3})$

$= \tan^{-1}(\sqrt{3}) - \pi + \cot^{-1}(\sqrt{3})$

$= \frac{\pi}{2} - \pi = -\frac{\pi}{2} \Rightarrow (B \to I)$

(C) $\cos^{-1}\left(\cos\frac{13\pi}{6}\right) = \cos^{-1}\left[\cos\left(2\pi + \frac{\pi}{6}\right)\right]$

$= \cos^{-1}\left(\cos\frac{\pi}{6}\right) = \frac{\pi}{6} \Rightarrow (C \to IV)$

(D) $\sin^{-1}\left(-\frac{1}{2}\right) = -\sin^{-1}\left(\frac{1}{2}\right) = -\frac{\pi}{6} \Rightarrow (D \to II)$

12. (d) Given $P(A) = \frac{1}{2}, P(B) = \frac{1}{3}, P(C) = \frac{1}{4}$

(Probability that at least one will solve the problem = 1 – (Probability that no will be able to solve the problem)

$= 1 - [1 - P(A)][1 - P(B)][1 - P(C)]$

$= 1 - \left(\frac{1}{2}\right)\left(\frac{2}{3}\right)\left(\frac{3}{4}\right) = \frac{3}{4}.$

13. (b) Given $R = \{(1, 1), (2, 2), (3, 3), (1, 2), (2, 3), (1, 3)\}$

R is reflexive because $(1, 1), (2, 2), (3, 3) \in R$

Now since $(1, 2) \in R$ but $(2, 1) \notin R \Rightarrow$ not symmetric.

Again now, since $(1, 2)$ and $(2, 3) \in R \Rightarrow (1, 3) \in R$.

Hence R is transitive.

14. (a) $\frac{dy}{dx} = (1 + x^2)(1 + y^2)$

$\Rightarrow \int \frac{dy}{1 + y^2} = \int (1 + x^2)\, dx$

$\Rightarrow \tan^{-1} y = x + \frac{x^3}{3} + C.$

15. (b) Given $\sqrt{1 - x^2} + \sqrt{1 - y^2} = a(x - y)$

Let $x = \sin A$ and $y = \sin B$

$\Rightarrow \sqrt{1 - \sin^2 A} + \sqrt{1 - \sin^2 B} = a(\sin A - \sin B)$

$\Rightarrow 2\cos\left(\frac{A+B}{2}\right)\cdot\cos\left(\frac{A-B}{2}\right)$

$= a\left[2\cos\frac{A+B}{2}\cdot\sin\left(\frac{A-B}{2}\right)\right]$

$\Rightarrow \tan\left(\frac{A-B}{2}\right) = \frac{1}{a} \Rightarrow (A - B) = 2\tan^{-1}\left(\frac{1}{a}\right)$

$\Rightarrow \sin^{-1} x - \sin^{-1} y = 2\tan^{-1}\left(\frac{1}{a}\right)$

Differentiating w.r.t. 'x', both side, we get -

$\Rightarrow \frac{1}{\sqrt{1-x^2}} - \frac{1}{\sqrt{1-y^2}}\frac{dy}{dx} = 0$

$\Rightarrow \frac{dy}{dx} = \frac{\sqrt{1-y^2}}{\sqrt{1-x^2}}$

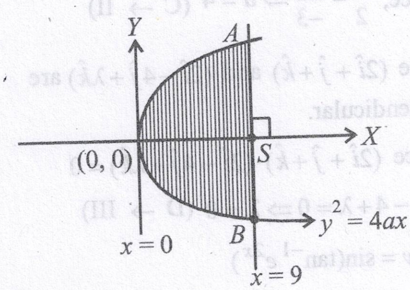
16. (a) Let $\alpha = 90°, \beta = 60°, \gamma = \theta$

Since $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

$\Rightarrow 0 + \left(\frac{1}{2}\right)^2 + \cos^2\theta = 1$

$\Rightarrow \cos\theta = \frac{\sqrt{3}}{2} \Rightarrow \theta = \frac{\pi}{6}$

17. (b) AB is latus rectum.



Required Area $= 2\int_{x=0}^{x=a} y\, dx$

$= 2\int_{x=0}^{x=a} \sqrt{4a}\cdot\sqrt{x}\, dx = \frac{8}{3}a^2$

18. (b) $|A| = \begin{vmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{vmatrix} = \cos^2\theta + \sin 2\theta = 1$

Now, $|AB| = |A||B| = (1)|B| = |B|$.

19. (c) $f[f(x)] = f(\sin x + x)$
$= \sin(\sin x + x) + (\sin x + x).$

20. **(d)** Let $I = \int \left(\frac{1+x+x^2}{1+x^2}\right) \cdot e^{\tan^{-1}x} dx$

Again, let $\tan^{-1}x = t \Rightarrow \frac{1}{1+x^2} dx = dt$

$I = \int (1+\tan t + \tan^2 t)e^t \, dt$

$= \int e^t \cdot \tan t \, dt + \int e^t \cdot \sec^2 t$

$= \int e^t \cdot \tan t \, dt + \left[e^t \tan t - \int e^t \cdot \tan t \, dt \right] + C$

$= x \, e^{\tan^{-1}x} + C.$

21. **(c)** Selling price of x items $= x \cdot (3x+5) = 3x^2 + 5x$

Cost price of x items $= x^2 + 5x$

Since there is no less, no profit.

Hence, $x^2 + 5x = 3x^2 + 5x \Rightarrow x = 0.$

22. **(b)** The direction ratio of given line is (3, 2, 6).

The direction ratio of the normal to the given plan is (2, 10, –11).

Hence required angle can be given by

$$\sin\theta = \frac{|(3)(2)+(2)(10)+(6)(-11)|}{\sqrt{(3)^2+(2)^2+(6)^2}\sqrt{(2)^2+(10)^2+(-11)^2}}$$

$\Rightarrow \sin\theta = \frac{8}{21} \Rightarrow \theta = \sin^{-1}\left(\frac{8}{21}\right).$

23. **(b)**

(A) Given $x_1 = 1, y_1 = 2, z_1 = 3$ and direction ratio is $a = 3$, $b = 2$, $c = -2$

Hence equation line is $\frac{x-1}{3} = \frac{y-2}{2} = \frac{z-3}{-2}$

Therefore, statement (A) is correct.

(B) Given $x_1 = 1, y_1 = 2, z_1 = 3$ and direction ratio is $a = 3$, $b = 5$, $c = 6$.

Hence equation of line is $\frac{x-1}{3} = \frac{y-2}{5} = \frac{z-3}{6}$.

Therefore statement is incorrect.

(C) Given $x_1 = 0, y_1 = 0, z_1 = 0$ and direction ratios are $a = 5 - 0 = 5$, $b = -2 - 0 = -2$, $c = 3 - 0 = 3$.

Hence equation of line is $\frac{x}{5} = \frac{y}{(-2)} = \frac{z}{3}$.

Therefore statement is correct.

(D) Given, $x_1 = 1$, $y_1 = 2$, $z_1 = 3$ and direction ratio of normal to the required plane $a = 2, b = 3, c = -1$.

Hence required plane is

$2(x-1) + 3(y-2) - 1(z-3) = 0$

Therefore statement is correct.

(E) Given intercepts are $a = 2, b = 3, c = 4$

Hence equation of plane $\frac{x}{2} + \frac{y}{3} + \frac{z}{4} = 1$

$\Rightarrow 6x + 4y + 3z = 12$

Therefore statement is wrong.

24. **(c)** Plotting option (c) will give the required result.

25. **(c)** Increased side $a_1 = a + \left(\frac{4a}{100}\right) = 1.04a$ meter

Hence required volume

$= a_1^3 = (1.04a)^3 = 1.12a^3 \text{ (meter)}^3$

26. **(b)** Given curve $y = -x^3 + 3x^2 + 9x - 27$

Slope $m = \frac{dy}{dx} = -3x^2 + 6x + 9$

Now for maxima or minima, $\frac{dm}{dx} = 0$

$\Rightarrow -6x + 6 = 0 \Rightarrow x = -1$

Now $\left(\frac{d^2m}{dx^2}\right)_{x=-1} = -6$

$\Rightarrow x = -1$ will be maxima point.

Hence maximum slope $= -3(1)^2 + 6(1) + 9 = 12.$

27. **(b)**

(A) $\int \frac{\sin x}{1+\cos x} dx = -\int \frac{-\sin x \, dx}{1+\cos x}$

$= -\log|1+\cos x| + C$ (A → III)

(B) $\int \frac{1}{1-\tan x} dx = \frac{1}{2}\int \frac{2\cos x}{\cos x - \sin x} dx$

$= \frac{1}{2}\int \frac{(\cos x - \sin x) + (\cos x + \sin x)}{(\cos x - \sin x)} dx$

$= \frac{1}{2}\int \left[1 + \frac{(\cos x + \sin x)}{(-\sin x + \cos x)}\right] dx$

$= \frac{x}{2} - \frac{1}{2}\log|\cos x - \sin x| + C$

(B → IV)

(C) $I = \int \frac{e^{\tan^{-1}x} dx}{1+x^2}$, let $\tan^{-1}x = t \Rightarrow \frac{1}{1+x^2} dx = dt$

$\Rightarrow I = \int e^t \, dt = e^t + C = e^{\tan^{-1}x} + C$ (C → I)

(D) $I = \int \frac{dx}{x(1+\log x)}$, let $1 + \log x = t \Rightarrow \frac{1}{x} dx = dt$

$I = \int \frac{dt}{t} = \log t + C = \log(\log x + 1) + C$ (D → II)

28. (b)

(a) A row matrix is a matrix with only one row.

(b) In a diagonal matrix all the elements except principal diagonal element is zero.

Hence option (b) is incorrect.

(c) A symmetric matrix is always a square matrix with condition $A^T = A$.

(d) A skew symmetric matrix have all diagonal element zero and $A^T = -A$.

29. (a) $A^2 = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \cdot \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$

$= \begin{bmatrix} \cos 2\theta & \sin 2\theta \\ -\sin 2\theta & \cos 2\theta \end{bmatrix}$

30. (c) $\dfrac{e^{\sin(\tan^{-1}x+\cot^{-1}x)}}{e^{\sin(\sin^{-1}x+\cos^{-1}x)}} = \dfrac{e^{\sin\left(\frac{\pi}{2}\right)}}{e^{\sin\left(\frac{\pi}{2}\right)}} = 1$

$\left\{\because \tan^{-1}x+\cot^{-1}x = \frac{\pi}{2},\ \sin^{-1}x+\cos^{-1}x = \frac{\pi}{2}\right\}$

31. (a) $-1 \le \sin 4x \le 1$

$\Rightarrow -1 \le -\sin 4x \le 1$

$\Rightarrow 3-1 \le 3-\sin 4x \le 3+1$

$\Rightarrow \frac{1}{4} \le \frac{1}{3-\sin 4x} \le \frac{1}{2}$

$\Rightarrow$ Range is $\left[\frac{1}{4}, \frac{1}{2}\right]$.

32. (b) Given $x+2y+3=0 \Rightarrow y = -\frac{1}{2}x - \frac{3}{2}$

Slope $m_1 = -\frac{1}{2}$, let m_2 be the slope perpendicular to $m_1 \Rightarrow m_1 m_2 = -1 \Rightarrow m_2 = 2$.

Given $y = x^2 - 2x - 3 \Rightarrow \frac{dy}{dx} = 2x - 2$

Since $m_2 = \frac{dy}{dx} \Rightarrow 2 = 2x - 2 \Rightarrow x = 2$

Hence $y = 2^2 - 2(2) - 3 = -3 \Rightarrow (x, y) = (2, -3)$

Hence equation of tangent passing through (2, –3) having slope $m_2 = 2$ is $y + 3 = 2(x-2)$

$\Rightarrow 2x - y = 7$.

33. (c) Given $2x\frac{dy}{dx} + y = 14x^3 \Rightarrow \frac{dy}{dx} + \frac{1}{2x}\cdot y = 7x^2$

If $= e^{\int \frac{1}{2x}dx} = e^{\frac{1}{2}\ln x} = \sqrt{x}$

Hence general solution is $(\text{IF})\cdot y = \int (7x^2)(\text{IF})dx + C$

$\Rightarrow \sqrt{x}\cdot y = \int 7x^{5/2}dx + C \Rightarrow y = 2x^3 + Cx^{-\frac{1}{2}}$.

34. (a) $\vec{a}, \vec{b}, \vec{c}$ are coplanar then $\left[\vec{a}\ \ \vec{b}\ \ \vec{c}\right] = 0$

$\Rightarrow \begin{vmatrix} 1 & -3 & 2 \\ 2 & 1 & -1 \\ 3 & 5 & -2\lambda \end{vmatrix} = 0 \Rightarrow 10\lambda + 10 = 0 \Rightarrow \lambda = -1.$

35. (d) $n = 7$\\node-d-220\Share Folder\36 yearwise NTA CUET Previous Year Solved Paper\CUET 2022-2023 (PCMB)\3_Math\Correction Pages

X(No. of heads) = 0, 1, 2, ... 7.

$p = \frac{1}{2}, q = \frac{1}{2}$

$\therefore\ P(X \ge 4) = P(X=4) + P(X=5) + P(X=6) + P(X=7)$

$= {}^7C_4\left(\frac{1}{2}\right)^4\left(\frac{1}{2}\right)^3 + {}^7C_5\left(\frac{1}{2}\right)^5\left(\frac{1}{2}\right)^2 + {}^7C_6\left(\frac{1}{2}\right)^6\left(\frac{1}{2}\right)^1 + {}^7C_7\left(\frac{1}{2}\right)^7\left(\frac{1}{2}\right)^0$

$= \left(\frac{1}{2}\right)^7\left[{}^7C_4 + {}^7C_5 + {}^7C_6 + {}^7C_7\right]$

$= \left(\frac{1}{2}\right)^7\left[\frac{7\cdot 6\cdot 5}{3\cdot 2\cdot 1} + \frac{7\cdot 6}{2\cdot 1} + 7 + 1\right]$

$= \left(\frac{1}{2}\right)^7[35+21+7+1] = \left(\frac{1}{2}\right)^7 \times 64 = \frac{1}{2}.$